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FUTURE UNIVERSE WITHOUT BIG-RIP PROBLEM

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ABSTRACT

In this paper, we showed that if cosmic dark energy behaves like a fluid with equation of state parameter $p = \omega\rho$ as well as modified chaplygin gas simultaneously then the big-rip problem does not arise even $\omega < -1$ and it is found that the scale factor is regular for all time

KEYWORDS: Dark Energy, Accelerated Universe, Bigf Rip

INTRODUCTION

A number of cosmological observations, such as Type Ia Supernovae [1, 2], cosmic microwave background (CMB) radiation [3], large scale structure (LSS) [4] shows that the universe is going through a phase of accelerated expansion. This phase is driven by a kind of unknown component, dubbed dark energy. The dark energy is usually described by an equation of state parameter $\omega = \frac{p}{\rho}$, the ratio of the homogeneous dark energy pressure over the energy density. For cosmic speed up, the value $\omega < -\frac{1}{3}$ is required. Moreover, observations from Wilkinson Microwave Anisotropy Probe (WMAP) indicates that the value of equation of state parameter $\omega \cong -1.10$ [5] and it means that our universe is dominated by phantom energy $(\omega < -1)$ [6, 7, 8, 9]. The phantom dominated universe end up with a finite time future singularity. The existence of this future singularity is often considered as a negative feature of phantom dark energy models so a great deal of effort has gone into constructing models with $\omega < -1$ to avoid this future singularity [10, 11, 12, 13, 14, 15]. In the braneworld scenario, Sahni and Shtanov has obtained well behaved expansion for the future universe without big rip problem with $w < -1$. They have shown that acceleration is a transient phenomenon in the current universe and the future universe will re-enter matter-dominated decelerated phase [16, 17]. It is found that general relativity based phantom model encounter "sudden future singularity" leading a divergent scale factor, energy density and pressure at finite time $t = t_s$. Thus the classical approach to phantom model exhibits big rip problem. For future singularity model, curvature invariant becomes very strong and energy density is very high near $t = t_s$ [18]. So, quantum effects should be dominated for $|t - t_s| < one\ unit\ of\ time$, like early universe [19, 20, 21] and it is shown that an escape from the big-smash is possible on making quantum corrections to energy density ρ and pressure p in Friedmann equations.

On the other hand many works can be found in the literature studying the implications of the use of the chaplygin gas as cosmological fluid. Its equation of state is defined as $p\rho = -A, (A > 0)$ and it is the only gas having super symmetry generalization [22, 23, 24]. Bertolami et al. [25] have found that generalized chaplygin gas is better fit for latest Supernova data.

In this paper, we consider the model, where dark energy behaves as modified chaplygin gas as well as fluid with equation of state $p = \omega\rho$ $(\omega < -1)$.The scale factor obtained here, does not possess future singularity. These are given in the section 'Field equation and its solution' and we conclude the paper with a brief discussion.

Field Equation and its Solution

The modified chaplygin gas is defined [26, 27, 28, 29, 30] by

$$p = (A-1)\rho - \frac{M}{\rho^{\alpha}} \tag{1}$$

where p is the pressure of the fluid, ρ its energy density, M, A, α are free parameters and $\alpha \geq -1$, $A > 1$

We consider the line element for the spatially homogeneous flat Friedmann-Robertson-Walker universe [31, 32]

$$dS^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2] \tag{2}$$

where *x*, *y*, *z* are the space co-ordinates, *t* is the time component and *a(t)* is the scale factor.

For this line element, the field equations are obtained as

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3} \tag{3}$$

And

$$2\dot{H} + 3\,H^2 = -p \tag{4}$$

The conservation equation for dark energy is given by

$$\dot{\rho} \;+\; 3H\,(\rho + p) = 0 \tag{5}$$

where dot denotes the differentiation with respect to 't'.

Using equation (1) in equation (5), it is obtained that

$$\frac{d\rho}{dt} + \frac{3}{a}\frac{da}{dt}\left[A\rho - \frac{M}{\rho^{\alpha}}\right] = 0 \tag{6}$$

This equation leads to

$$A\rho^{1+\alpha} = M + (A\rho_0^{1+\alpha} - M)\left(\frac{a_0}{a}\right)^{3A(1+\alpha)} \tag{7}$$

where ρ_0 and a_0 are the present values of energy density and scale factor at the present time t_0.

In the present model, it is assumed that the dark energy behaves like modified chaplygin gas, obeying equation (1) as well as fluid with equation of state

$$p = \omega\rho \tag{8}$$

with $\omega < -1$ simultaneously.

Equation (1) and (8) yield as

$$\omega(t) = A - 1 - \frac{M}{\rho^{1+\alpha}} \tag{9}$$

At the present time t_0, the equation (9) leads to

$$(A-1-\omega_0)\rho_0^{1+\alpha} = M \tag{10}$$

with $\omega_0 = \omega(t_0)$.

From equations (7) and (10), it is obtained that

$$A\rho^{1+\alpha} = (A-1-\omega_0)\rho_0^{1+\alpha} + (1+\omega_0)\rho_0^{1+\alpha}\left(\frac{a_0}{a}\right)^{3A(1+\alpha)} \tag{11}$$

In the homogeneous model of the universe, a scalar field $\varphi(t)$ with potential $V(\varphi)$ has energy density

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \tag{12}$$

And pressure

$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \tag{13}$$

From equations (12) and (13) it is obtained that

$$\dot{\varphi}^2 = \rho_\varphi + p_\varphi \tag{14}$$

Using equations (1), (8), (10), equation (14) reduces to

$$\dot{\varphi}^2 = \frac{A\rho^{1+\alpha} - M}{\rho^\alpha} = \frac{A\rho^{1+\alpha} - (A-1-\omega_0)\rho_0^{1+\alpha}}{\rho^\alpha} \tag{15}$$

Equations (11) and (15) lead to

$$\dot{\varphi}^2 = \frac{(1+\omega_0)\rho_0\left(\frac{a_0}{a}\right)^{3A(1+\alpha)}}{\left[\frac{A-1-\omega_0}{A} + \left(\frac{1+\omega_0}{A}\right)\left(\frac{a_0}{a}\right)^{3A(1+\alpha)}\right]^{\frac{\alpha}{1+\alpha}}} \tag{16}$$

This equation shows that $\dot{\varphi}^2 > 0$ (positive kinetic energy) for $1+\omega_0 > 0$ and $\dot{\varphi}^2 < 0$ (negative kinetic energy) for $1+\omega_0 < 0$. $1+\omega_0 > 0$ and $1+\omega_0 < 0$ are representing the case of quintessence and phantom fluid dominated universe respectively. Similar results are obtained by Hoyle and Narlikar in C-field (a scalar called creation field) with negative kinetic energy for steady state theory of universe [33, 34].

Thus, it is shown that dual behavior of dark energy fluid, obeying equations (1) and (8) is possible for scalars, frequently used for cosmological dynamics. So, this assumption is not unrealistic.

Now from equations (3) and (11), the Friedmann equation with dominance of dark energy having double fluid behavior is

$$H^2 = H_0^2\Omega_0\left[\frac{A-1+|\omega_0|}{A} + \frac{1-|\omega_0|}{A}\left(\frac{a_0}{a}\right)^{3A(1+\alpha)}\right]^{\frac{1}{1+\alpha}} \tag{17}$$

where $|\omega_0| > 1$, H_0 is the present value of Hubble's constant and $\Omega_0 = \frac{\rho_0}{\rho_{cr,0}}$ with $\rho_{cr,0} = \frac{3H_0^2}{8\pi G}$ (G being the Newtonian gravitational constant).

Neglecting higher powers of $\frac{1-|\omega_0|}{A-1+|\omega_0|}\left(\frac{a_0}{a}\right)^{3A(1+\alpha)}$, equation (17) may be written as

$$\frac{\dot{a}}{a} \approx H_0\sqrt{\Omega_0}\left\{\frac{A-1+|\omega_0|}{A}\right\}^{\frac{1}{2(1+\alpha)}}\left[1+\frac{1-|\omega_0|}{2(1+\alpha)(A-1+|\omega_0|)}\left(\frac{a_0}{a}\right)^{3A(1+\alpha)}\right] \tag{18}$$

Integrating equation (18), we obtain

$$a(t) = \frac{a_0}{\{2(1+\alpha)\}^{\frac{1}{3A(1+\alpha)}}}\left[\left\{\frac{1-|\omega_0|}{A-1+|\omega_0|}+2(1+\alpha)\right\}e^{H_0\sqrt{\Omega_0}\left\{\frac{A-1+|\omega_0|}{A}\right\}^{\frac{1}{2(1+\alpha)}}3A(1+\alpha)(t-t_0)}-\frac{1-|\omega_0|}{A-1+|\omega_0|}\right]^{\frac{1}{3A(1+\alpha)}} \tag{19}$$

Equation (19) shows that $a(t) \to \infty$ as $t \to \infty$ and observations of Supernova Ia [1, 2], WMAP [3] also supported this result. Therefore this model is free from finite time future singularity.

The horizon distance for this case is obtained as

$$d_H(t) \simeq \frac{3A(1+\alpha)a(t)}{a_0}\left[\frac{2(1+\alpha)}{\frac{1-|\omega_0|}{A-1+|\omega_0|}+2(1+\alpha)}\right]^{\frac{1}{3A(1+\alpha)}} exp\left[H_0\sqrt{\Omega_0}\left\{\frac{A-1+|\omega_0|}{A}\right\}^{\frac{1}{2(1+\alpha)}} t\right] \tag{20}$$

From equations (19) and (20), it is clear that

$d_H(t) > a(t)$

So, horizon grows more rapidly than the scale factor.

In this case, the Hubble distance is given by

$$H^{-1} = \frac{1}{H_0\sqrt{\Omega_0}\left\{\frac{A-1+|\omega_0|}{A}\right\}^{\frac{1}{2(1+\alpha)}}}\left[1-\frac{1-|\omega_0|}{2(1+\alpha)(A-1+|\omega_0|)}\left(\frac{a_0}{a}\right)^{3A(1+\alpha)}\right] \tag{21}$$

This shows that $H^{-1} \to \frac{1}{H_0\sqrt{\Omega_0}\left\{\frac{A-1+|\omega_0|}{A}\right\}^{\frac{1}{2(1+\alpha)}}} \neq 0$ at infinite time. It means that the galaxies will not disappear when $t \to \infty$.

Moreover, the equation (11) may be written as

$$\rho = \rho_0\left[\frac{A-1+|\omega_0|}{A}+\frac{1-|\omega_0|}{A}\left(\frac{a_0}{a(t)}\right)^{3A(1+\alpha)}\right]^{\frac{1}{1+\alpha}} \tag{22}$$

And this shows that $\rho \to \rho_0\left(\frac{A-1+|\omega_0|}{A}\right)^{\frac{1}{1+\alpha}}$ (finite) when $t \to \infty$ (since $t \to \infty, a(t) \to \infty$).

Thus the energy density increases with time.

CONCLUSIONS

In this paper, it is found that if dark energy behaves as modified chaplygin gas and fluid with equation of state $p = \omega\rho$, it is possible to get accelerated growth of a (t) for time $t_0 < t < \infty$ with no future singularity. Here also, the energy density increases with time, contrary to other phantom models having future singularity at $t = t_s$ [6, 7]. In Refs [18, 19, 20, 21], for models with future singularity escape from catastrophic situation, is demonstrated using quantum corrections in field equations near $t = t_s$. But the present model for phantom cosmology without big rip is explored by using classical approach.

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